An example of medical treatment optimization under model uncertainty

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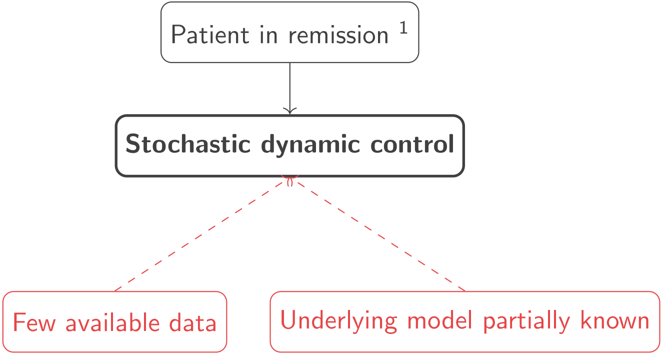
June 7, 2023

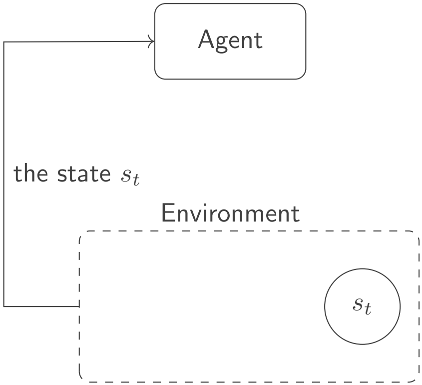
A medical context



How can these issues be addressed in a simplified problem ?

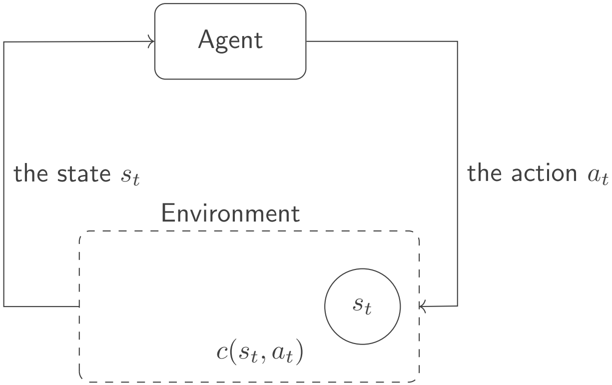
[1] Data from IUC Oncopole, Toulouse, and CRCT, Toulouse, France

Markov Decision Process (MDP)



- $s \in \mathcal{S}$ the state space
- $a \in \mathcal{A}$ the action space
- $\mathcal{P}$ the transition matrix
- $c(s_t, a_t)$ the cost function

Markov Decision Process (MDP)



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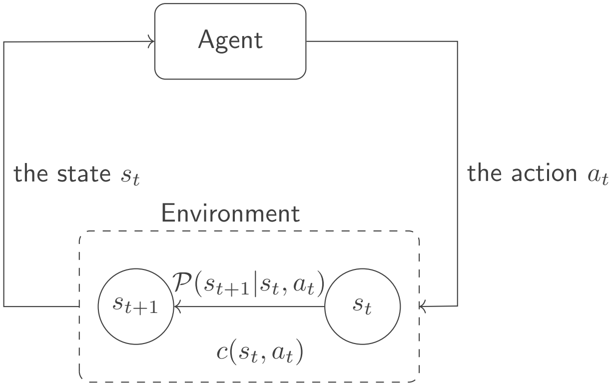

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State transition

The transition matrix is partially known

Table: **Transition matrix when patient has no treatment ($a = \emptyset$).**

$s_t \backslash s_{t+1}$	$(0,0,0)$	$(1,0,1)$	$(1,0,2)$	$(1,1,1)$	$(1,1,2)$	$(1,2,1)$	$(1,2,2)$	$(1,3,1)$	$(1,3,2)$	$(2,4,0)$
$(0,0,0)$	$p^{\emptyset}_{(0,0,0)}$	$p^{\emptyset}_{(1,0,1)}$	$p^{\emptyset}_{(1,0,2)}$	0	0	0	0	0	0	0
$(1,0,1)$	0	0	0	**1**	0	0	0	0	0	0
$(1,0,2)$	0	0	0	0	0	0	**1**	0	0	0
$(1,1,1)$	0	0	0	0	0	**1**	0	0	0	0
$(1,1,2)$	0	0	0	0	0	0	0	0	**1**	0
$(1,2,1)$	0	0	0	0	0	0	0	**1**	0	0
$(1,2,2)$	0	0	0	0	0	0	0	0	0	**1**
$(1,3,1)$	0	0	0	0	0	0	0	0	0	**1**
$(1,3,2)$	0	0	0	0	0	0	0	0	0	**1**
$(2,4,0)$	0	0	0	0	0	0	0	0	0	**1**

State transition

The transition matrix is partially known

Table: **Transition matrix when patient has treatment ($a = \rho$).**

$s_t \backslash s_{t+1}$	$(0,0,0)$	$(1,0,1)$	$(1,0,2)$	$(1,1,1)$	$(1,1,2)$	$(1,2,1)$	$(1,2,2)$	$(1,3,1)$	$(1,3,2)$	$(2,4,0)$
$(0,0,0)$	$p^{\rho}_{(0,0,0)}$	$p^{\rho}_{(1,0,1)}$	$p^{\rho}_{(1,0,2)}$	0	0	0	0	0	0	0
$(1,0,1)$	**1**	0	0	0	0	0	0	0	0	0
$(1,0,2)$	**1**	0	0	0	0	0	0	0	0	0
$(1,1,1)$	**1**	0	0	0	0	0	0	0	0	0
$(1,1,2)$	**1**	0	0	0	0	0	0	0	0	0
$(1,2,1)$	0	0	0	**1**	0	0	0	0	0	0
$(1,2,2)$	0	0	0	0	**1**	0	0	0	0	0
$(1,3,1)$	0	0	0	0	0	0	0	**1**	0	0
$(1,3,2)$	0	0	0	0	0	0	0	0	**1**	0
$(2,4,0)$	0	0	0	0	0	0	0	0	0	**1**

The list of costs:

- Treatment: 300
- Disease 1: 200
- Disease 2: 300
- Death: 1000

Policy π

Let $f : \mathcal{S} \to \mathcal{A}$ for all $s \in \mathcal{S}$ is a decision rule.
A sequence of decision rules $\pi = (f_0, f_1, \ldots, f_{H-1})$ is a policy.

Policy cost

$$J_H(\pi, s) = \mathbb{E}[\sum_{t=0}^{H-1} c(s_t, a_t) | \pi, s]$$

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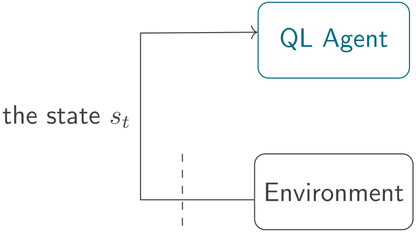
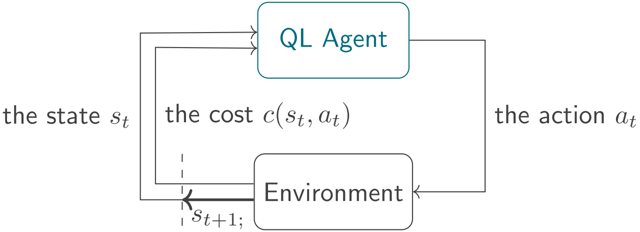
Optimization criterion

$$V^\star(s_t) = \min_{a \in \mathcal{A}}[c(s_t, a_t) + \sum_{s_{t+1} \in \mathcal{S}} \mathcal{P}(s_{t+1}|s_t, a_t)V^\star(s_{t+1})]$$

A model-free method

Q-learning[2,3] algorithm





[2] Christopher J. C. H. Watkins and Peter Dayan (May 1992). "Q-learning". In: *Mach. Learn.* 8.3, pp. 279–292. ISSN: 1573-0565. DOI: 10.1007/BF00992698.

[3] VP Vivek and Dr. Shalabh Bhatnagar (Aug. 2022). "Finite Horizon Q-learning: Stability, Convergence, Simulations and an application on Smart Grids". In: *arXiv:2110.15093v3*. DOI: 10.48550/arXiv.2110.15093. eprint: 2110.15093v3.

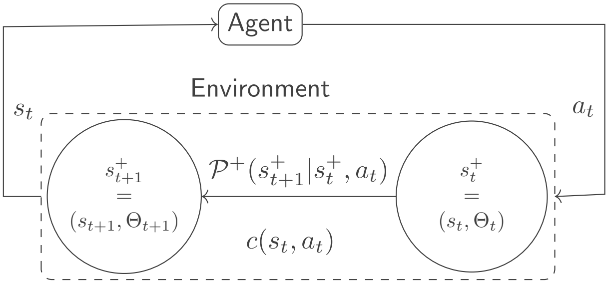
A bayesian approach

$s_t \backslash s_{t+1}$	$(0,0,0)$	$(1,0,1)$	$(1,0,2)$	$(1,1,1)$	$(1,1,2)$	$(1,2,1)$	$(1,2,2)$	$(1,3,1)$	$(1,3,2)$	$(2,4,0)$
$(0,0,0)$	$p^{\emptyset}_{(0,0,0)}$	$p^{\emptyset}_{(1,0,1)}$	$p^{\emptyset}_{(1,0,2)}$	0	0	0	0	0	0	0

Remark:

- $P(.|s=(0,0,0), a=\emptyset) \sim \mathcal{M}(p^{\emptyset}_{(0,0,0)}, p^{\emptyset}_{(1,0,1)}, p^{\emptyset}_{(1,0,2)})$
- Conjugate distribution : $f(p^{\emptyset}|\Theta^{\emptyset}) \sim \mathcal{D}(\theta^{\emptyset}_{(0,0,0)}, \theta^{\emptyset}_{(1,0,1)}, \theta^{\emptyset}_{(1,0,2)})$
- $f(p^{\emptyset}|\Theta^{\emptyset}) \propto \prod_{s_{t+1} \in \mathcal{S}} p_{s_{t+1}}^{\emptyset, \theta^{\emptyset}_{s_{t+1}} - 1}$

Bayes-Adaptive Markov Decision Process (BAMDP[4])



- $s^+ \in \mathcal{S}^+$ the hyper-state space
- $\mathcal{P}^+$ the transition matrix
- $\Theta_{t+1} = \Theta_t + \Delta^{a_t}_{s_{t+1}}$, with

$$\Delta^{a_t}_{s_{t+1}} = \begin{cases} 1 & \text{if } (s = (0, 0, 0), a_t, s_{t+1}), \\ 0 & \text{else.} \end{cases}$$

[4] Michael O'Gordon Duff (2002). "Optimal learning: Computational procedures for Bayes -adaptive Markov decision processes". PhD thesis. University of Massachusetts Amherst.

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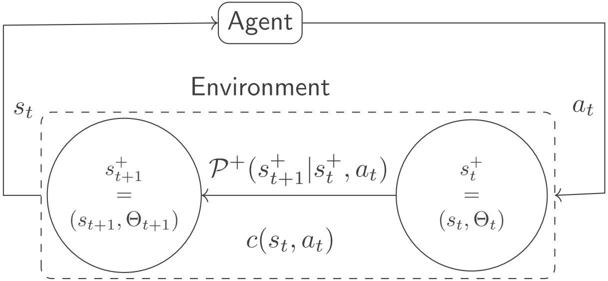

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Optimization criterion

$$V^\star(s_t, \Theta_t) = \min_{a \in \mathcal{A}} [c(s_t, a_t) + \sum_{s^+_{t+1} \in \mathcal{S}^+} \mathcal{P}^+(s^+_{t+1}|s^+_t, a_t) V^\star(s_{t+1}, \Theta_{t+1})]$$

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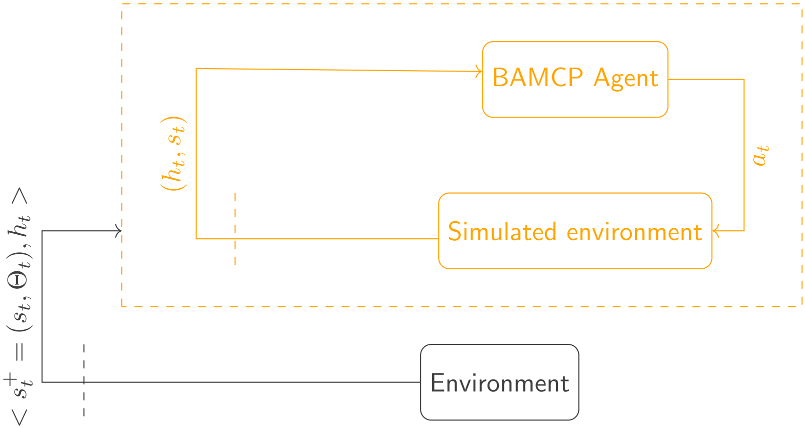
A model-based method

Bayes-Adaptive Monte-Carlo Planning (BAMCP[5])

with $h_t = (s_0, a_0, s_1, \dots, s_{t-1}, a_{t-1})$,



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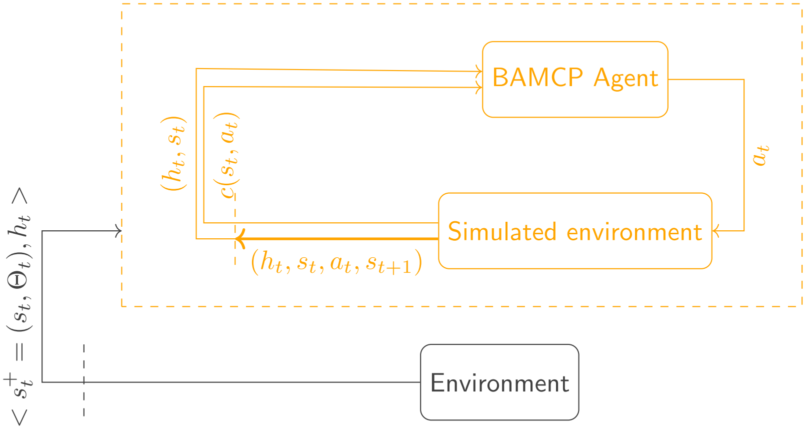

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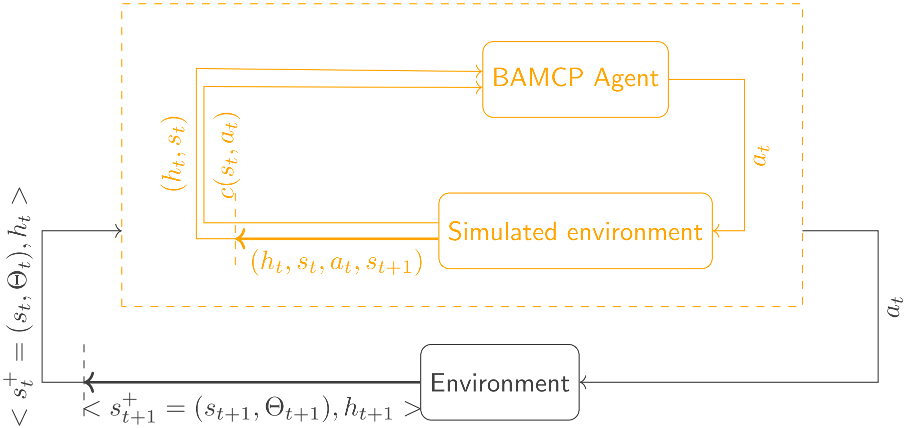
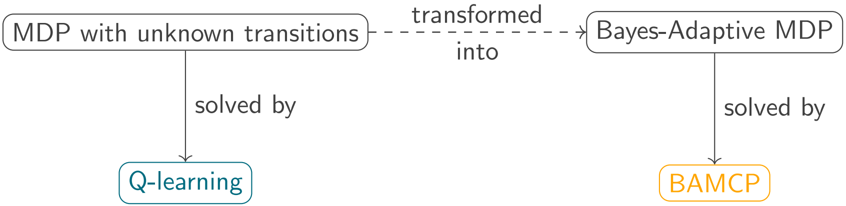


Results

The optimal policy exact cost: 888.89

Simulated patients	Q-learning		BAMCP	
	Cost	Time	Cost	Time
10^2	1427.06 ± 1.05	0.07 sec	1377.62 ± 1.21	15.41 min
10^3	936.96 ± 0.70	2.48 min	1340.92 ± 1.04	17.86 min
10^4	936.93 ± 0.70	4.17 min	NC	4 days
10^6	891.6 ± 0.68	10.21 min	NC	1.5 years

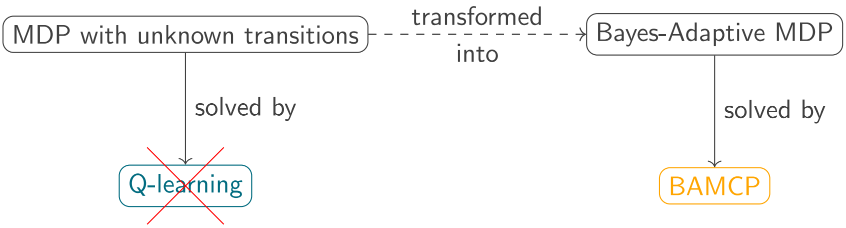
Conclusion



- Mathemathical framework
- Model-free method
- Model-based method

Model-free methods don't work because we don't have enough to interact with.

Conclusion



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Perspectives

MDP model	$\rightarrow$	PDMP[6] model
Finite state space	$\rightarrow$	Continuous state space
Markovian	$\rightarrow$	Semi-Markovian
Complete observations	$\rightarrow$	Hidden observations

Unlike model-free methods and deep reinforcement learning, **bayesian approaches** do not require as much interaction with the environment.

[6] Mark H. A. Davis (1984). "Piecewise-Deterministic Markov Processes: A General Class of Non-Diffusion Stochastic Models". In: *Journal of the Royal Statistical Society Series B (Methodological)* 46, pp. 353–376.