Numerical method for optimal strategies for impulse control of piecewise deterministic Markov process

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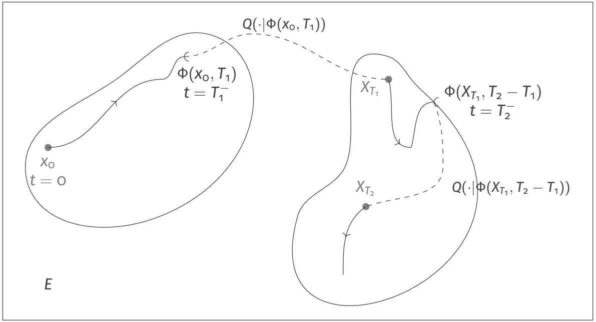
Move randomly from one deterministic regime to another.



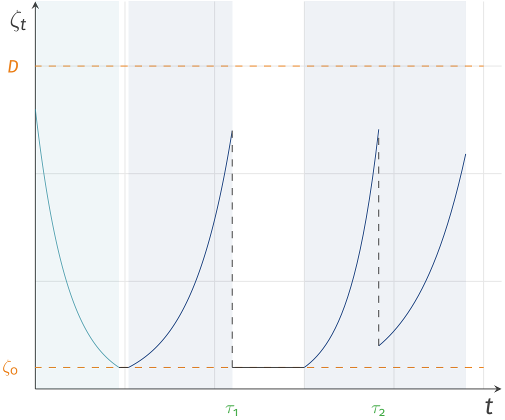
The process $X = (X_t)_{t \geq 0}$ is defined on the state space E by 3 local characteristics.

Local characteristics

- Flow Φ (deterministic motion of the process)
- Jump Intensity λ (occurence of random jumps)
- Markov Kernel Q (post-jump localisation)

Impulse control for PDMP
Example

Select new starting point for the process at interventions to minimize a cost function.



Strategy

$$\mathcal{S} = (\tau_n, R_n)_{n\geq 1}$$

- τ_n intervention dates
- R_n new positions after intervention

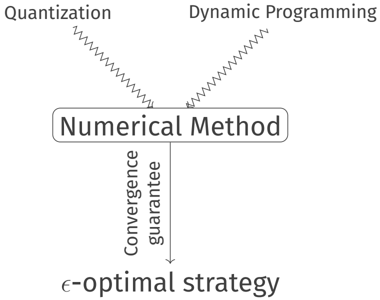
$$J^{\mathcal{S}}(x) = E_x^{\mathcal{S}}\left[\int_0^\infty e^{-\alpha s} \underbrace{f(X_s)}_{\text{running cost}} ds + \sum e^{-\alpha\tau_i} \underbrace{c(X_{\tau_i}, X_{\tau_i+})}_{\text{intervention cost}}\right]$$

$$\underbrace{V(x) = \inf_{\mathcal{S}\in\mathbb{S}} J^{\mathcal{S}}(x)}_{\text{Value function}}$$

Resolution

ϵ-optimal Strategy

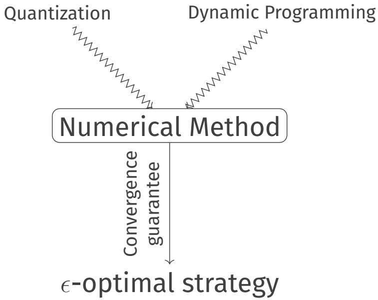
$$V(\mathcal{S}_\epsilon, x) \leq V(x) + \epsilon$$



Challenges :

- Numerical
- Mathematical

Conclusion



Numerical method :

- Get ϵ-optimal strategy
- Convergence guarantee

Limits :

- Process fully observed (including jump dates)
- Process fully known

Perspectives

Ongoing work

Solve **partially observed** PDMP with a **partially known model** using **Deep Reinforcement Learning Algorithm**.

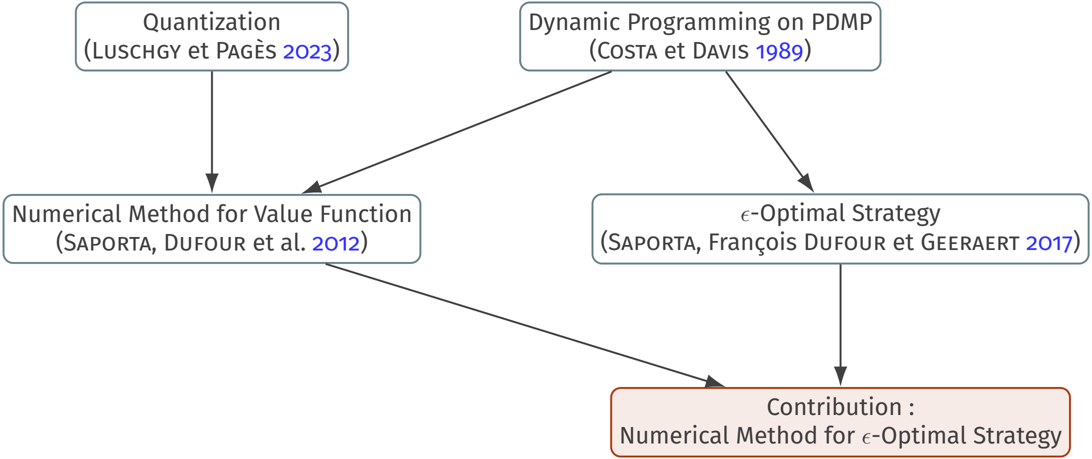
Limits

Neural network resolution methods are difficult to interpret and the cost function is complicated to calibrate.

Current work

With this method, we aim to :

- **Easily test multiple cost functions**.
- **Compare the resulting strategies** and choose the best one based on various objectives.


Quantization
(LUSCHGY et PAGÈS 2023)
Dynamic Programming on PDMP
(COSTA et DAVIS 1989)
Numerical Method for Value Function
(SAPORTA, DUFOUR et al. 2012)
ϵ-Optimal Strategy
(SAPORTA, François DUFOUR et GEERAERT 2017)
Contribution :
Numerical Method for ϵ-Optimal Strategy

References

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- Luschgy, Harald et Gilles Pagès (2023). Marginal and Functional Quantization of Stochastic Processes. Cham, Switzerland : Springer Nature Switzerland. ISBN : 978-3-031-45464-6.
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